O K L A H O M A   S T A T E   U N I V E R S I T Y

S C H O O L   O F   E L E C T R I C A L   A N D   C O M P U T E R   E N G I N E E R I N G

ECEN 5713 Linear System
Spring 1999
Midterm Exam #2

Name : ______________________________

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<u>Problem 1</u>:

For

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix},$$

determine the rank and nullity of the above linear operator, A ? And find a basis for the range space and the null space of the linear operator, A, respectively ?

Problem 2:

Extend the set

$$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 4 \end{bmatrix}$$

to form a basis in $(\Re^4, \Re)$.

Problem 3:

Show that

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$$

span the same subspace V of $(\Re^3, \Re)$.

Problem 4:

Find the relationship between the two bases $\{v_1, v_2, v_3\}$ and $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$, where

$$v_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \bar{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \bar{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \bar{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Also determine the representation of the vector $e_2^T = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$ with respect to both of the above bases.

Problem 5:

A vector space, *V*, is spanned by v_1, v_2, v_3 given as

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -5 \\ 1 \\ 1 \\ 5 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ 2 \\ 2 \\ 1 \end{bmatrix}.$$

Determine the orthogonal complement space of *V*, $V^\perp$, and find a basis and dimension of $V^\perp$.

For $x = \begin{bmatrix} 0 & 3 & 3 & 0 \end{bmatrix}^T$, find its direct sum representation of $x = x_1 \oplus x_2$, such that $x_1 \in V$, $x_2 \in V^\perp$.